

# A Multifractal Description of Wind Speed Records

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## Abstract

In this paper, a systematic analysis of hourly wind speed data obtained from four potential wind generation sites in North Dakota is conducted. The power spectra of the data exhibited a power law decay characteristic of  $1/f^\alpha$  processes with possible long range correlations. The temporal scaling properties of the records were studied using multifractal detrended fluctuation analysis *MF DFA*. It is seen that the records at all four locations exhibit similar scaling behavior which is also reflected in the multifractal spectrum determined under the assumption of a binomial multiplicative cascade model.

## 1 Introduction

Wind energy is a ubiquitous resource and is a promising alternative to meet the increased demand for energy in recent years. Unlike traditional power plants, wind generated power is subject to fluctuations due to the intermittent nature of wind. The irregular waxing and waning of wind can lead to significant mechanical stress on the gear boxes and result in substantial voltage swings at the terminals, [1]. Therefore, it is important to build suitable mathematical techniques to understand the temporal behavior and dynamics of wind speed for the purposes of modeling, prediction, simulation and design. Attempts to identify the features of wind speed time series data were described in [2] and [3]. To our knowledge, the first paper to bring out an important feature of wind speed time series was [4]. In [4], the authors examined long term records of hourly wind speeds in Ireland and pointed out that they exhibited what is known as long term dependence. Seasonal effects, spatial correlations and temporal dependencies were incorporated to build suitable estimators. Evidence for the presence of long memory correlations was provided by inspecting the periodogram of the residuals from a fitted and autoregressive model of order nine i.e. AR(9) [4].

For the pursuit of wind energy development in the state of North Dakota which has an estimated potential of 250 GW, [5], several wind monitoring stations have been set up where the hourly average wind speeds along with meteorological variables are being recorded. As the power developed by the wind turbines is proportional to the cube of the wind speed, the recorded data can be used to assess the wind power potential of the site. In addition, an analysis of the recorded data can be useful in understanding the nature of the fluctuations in power developed by the wind turbine. Conventionally, the statistical modeling of wind speed data has been done by using Weibull and Rayleigh probability distribution functions, [6]. While these methods are useful in providing estimates of wind generated energy yield, they do not explicitly bring out the nature of the temporal variations in wind speed and thus, wind power. Complex motions in the atmosphere tend to render the wind speed distinctly non-stationary and intermittent. Traditional power spectral (or equivalently the auto correlation function) analyses are restricted in their scope for the present study due to their susceptibility to non-stationarity. Power spectral techniques have been useful successfully to detect possible long-range correlations of the form of the form  $S(f) \sim 1/f^\beta$ , [7]. Long range correlations generally indicate that temporally well separated samples of the time series are correlated with each other and indicative of self-similar (self-affine) behavior. Self similar time series can be characterized by

$$y(t) \equiv a^\alpha y(t/\alpha) \quad (1)$$

where the  $\equiv$  in Eqn.(1) denotes that both sides of the equation have identical statistical properties. The exponent  $\alpha$  in Eqn.(1) is called the self-similarity parameter, or the scaling exponent. Values of  $\alpha$  in the range  $(0, 0.5)$  characterize anti-persistence, whereas those in the range  $(0.5, 1)$  characterize persistent long range correlations with  $\alpha = 0.5$  representing uncorrelated noise. The exponent ( $\beta$ ) estimated from the power spectrum ( $S(f) \sim 1/f^\beta$ ) is related to  $\alpha$  as  $\beta = 2\alpha - 1$ . The temporal trace and corresponding power spectrum of one of the representative records is shown in Fig. 1.

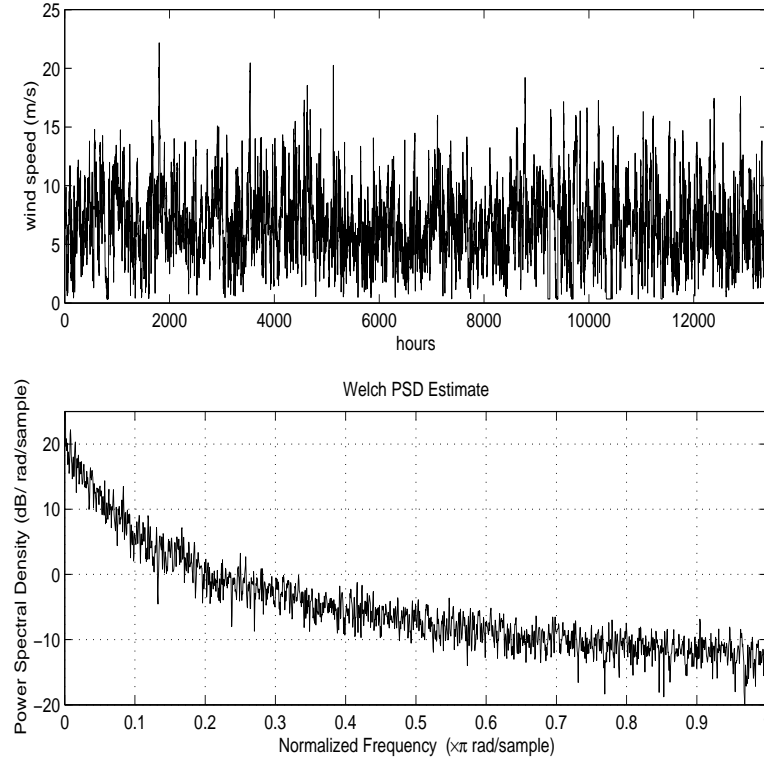


Figure 1: Temporal trace and Power spectrum of a representative record

Examining the power spectrum of the wind speed indicates a power-law decay of the form  $S(f) \sim 1/f^\beta$ . The auto-correlation functions (ACF) decay slowly to zero and the first zero crossing of the ACFs occur at lags of 61,56,60 and 57 respectively for the four data sets. Such features are characteristic of statistically self similar processes with well defined power law correlations, [7].

Estimating the Hurst exponent ( $H$ ) from the given data is an alternate effective way to determine the nature of the correlations in it [8]. Hurst exponents have been successfully used to quantify long range correlations in plasma turbulence [9], [10], finance [11], [12], network traffic, [13] and physiology, [14]. Long range correlations are said to exist if  $0.5 < H < 1$  (see [15], [16] for details). There are several methods such as the variance method, re-scaled range (R/S) analysis, Whittle estimator and wavelet based methods to estimate the Hurst exponent. Hurst estimators are susceptible to such artifacts such as polynomial trends which cannot be ruled out in experimental data and therefore, they may give spurious results. Recently the Detrended Fluctuation Analysis (DFA), [17] and its extensions have been proposed as an alternate effective technique to determine possible long range correlations in data sets obtained from a diverse settings, [18], [19], [20], [21].

It should be noted that the techniques listed above can only extract a single scaling exponent from a time series. However, it is possible that the given process may be governed by more than one scaling exponents, in which case a single scaling exponent would be unable to capture the complex dynamics inherent in the data. Therefore,

these methods are appropriate only for the analysis of monofractal signals which have uniform scaling properties throughout the signal which can be characterized by a single exponent. On the other hand, multifractal signals are far more complex than monofractal signals and require more than one (theoretically infinite) exponent to characterize their scaling properties, [14].

In the present study, the wind speed time series data is studied using a fairly robust and powerful technique called *Multifractal Detrended Fluctuation Analysis* (MFDFA), [22]. The method provides a systematic means to identify and more importantly quantify the multiple scaling exponents in the data [22]. The scaling exponents of the data are estimated under the assumption of a binomial multiplicative cascade model. This is carried out on the wind speed data acquired from four spatially separated monitoring locations in North Dakota. The paper is organized as follows. In Sec.2, the acquisition of wind speed data is described. In Sec. 3, a brief description of multifractal detrended fluctuation analysis is provided. In Sec.4, the results of *MFDFA* and the calculation of the multifractal spectrum is provided. The conclusions are summarized in Sec.5.

## 2 Data Acquisition

The hourly averaged wind speeds at four geographically well separated locations in North Dakota were recorded by means of a conventional cup type anemometers located at a height of 20 m. The co-ordinates of the map are provided in Table 1. From Fig.2, it can be seen that the locations are spatially well separated across North Dakota. The wind speed data to be discussed were recorded over a time frame ranging from 11/29/2001 to 07/28/2003.

Table 1: Co-ordinates of the wind monitoring sites

Station	Latitude	Longitude	Elvn (ft)
Site 1	N 47 27.84'	W 99 8.18'	1570
Site 2	N 46 13.03'	W 97 15.10'	1070
Site 3	N 48 52.75'	W 103 28.4'	2270
Site 4	N 46 12.67'	W 103 13.07'	2880



Figure 2: A map of the wind monitoring sites

## 3 Multifractal Detrended Fluctuation Analysis (MFDFA)

MFDFA, [22] a generalization of DFA has been shown to reliably extract more than one scaling exponent from a time series. A brief description of the algorithm is provided in Section III, for completeness. A detailed explanation can be found elsewhere [22]. Consider a time series  $\{x_k\}, k = 1 \dots N$ . The MFDFA algorithm consists of the following steps.

1. The series  $\{x_k\}$  is integrated to form the integrated series (also called “profile”)  $\{y_k\}$  given by

$$y(k) = \sum_{i=1}^{i=k} [x(i) - \bar{x}] \quad k = 1, \dots, N \quad (2)$$

where  $\bar{x}$  represents the average value.

2. The series  $\{y_k\}$  is divided in to  $n_s$  non-overlapping boxes of equal lengths where  $n_s = \text{int}(N/s)$ . To accommodate the fact that some of the data points may be left out, the procedure is repeated from the other end of the data set [22].
3. The local polynomial trend  $y_v$  with order  $v$  is fit to the data in each box, the corresponding variance is given by

$$F^2(v, s) = \left\{ \frac{1}{s} \sum_{i=1}^{i=s} \{y[N - (v - n_s)s + i] - y_v(i)\}^2 \right\} \quad (3)$$

for  $v = 1, \dots, n_s$ . Polynomial detrending of order  $m$  is capable of eliminating trends up to order  $m-1$ . [23]

4. The  $q$ th order fluctuation function is calculated from averaging over all segments.

$$F_q(s) = \left\{ \frac{1}{2n_s} \sum_{i=1}^{i=2n_s} [F^2(v, s)]^{q/2} \right\}^{1/q} \quad (4)$$

In general, the index  $q$  can take any real value except zero.

5. Step 3 is repeated over various time scales  $s$ . The scaling of the fluctuation functions  $F_q(s)$  versus the time scale  $s$  is revealed by the log-log plot.
6. The scaling behavior of the fluctuation functions is determined by analyzing the log-log plots  $F_q(s)$  versus  $s$  for *each*  $q$ . If the original series  $\{x_k\}$  is power-law correlated, the fluctuation function will vary as

$$F_q(s) \sim s^{h(q)} \quad (5)$$

### 3.1 Multifractal Analysis

In the MFDFA procedure (Section III), the exponent  $h(q)$  describes the scaling behavior of the  $q$ th order fluctuation function. For positive values of  $q$ ,  $h(q)$  describes the scaling behavior of segments with large fluctuations while those of negative values of  $q$ , describe scaling behavior of segments with small fluctuations, [22]. For stationary time series, the exponent  $h(2)$  is identical to the Hurst exponent. Thus the exponent  $h(q)$  is called as the generalized Hurst exponent, [22]. For monofractal time series which are characterized by a single exponent over all scales,  $h(q)$  is independent of  $q$ , whereas for a multifractal time series,  $h(q)$  varies with  $q$ . This dependence is considered to be a characteristic property of multifractal processes, [22].

A multifractal description can also be obtained from considering partition functions ( see [24] for details)

$$Z_q(s) = \sum_{v=1}^{v=n_s} |y_{vs} - y_{(v-1)s}|^q \sim s^{\tau(q)} \quad (6)$$

where  $\tau(q)$  is the Renyi exponent. A linear scaling of  $\tau(q)$  with  $q$  is characteristic of a monofractal data, whereas a nonlinear scaling is indicative of multifractal behavior. The  $h(q)$  obtained from MFDFA is related to the the Renyi exponent  $\tau(q)$  by

$$qh(q) = \tau(q) + 1 \quad (7)$$

Therefore, another way to characterize a multifractal series is the multifractal spectrum  $f(\alpha_h)$  defined by, [7]

$$\alpha_h = \frac{d\tau(q)}{dq}, \quad f(\alpha_h) = q\alpha_h - \tau(q) \quad (8)$$

In (9),  $\alpha_h$  is the Hölder exponent which characterizes the singularities (cusps, ridges, chirps, spikes) in a process  $X(t)$  at time  $t$ , [25]. The multifractal spectrum  $f(\alpha_h)$  describes the singularity content of the process, i.e. the dimension of the set of times  $t$  where  $\alpha_h = \alpha$ , [25]. Substituting for  $\tau(q)$  from Eqn.(7), we get, [22]

$$\alpha_h = h(q) + q \frac{dh(q)}{dq}, \quad f(\alpha_h) = q[\alpha_h - h(q)] + 1 \quad (9)$$

The generalized exponents  $h(q)$  in Eqn.(9) can be estimated by the formula

$$h(q) = \frac{1}{q} - \frac{\ln(a^q + b^q)}{q \ln 2} \quad (10)$$

assuming a binomial multiplicative cascade model [7] which has served as one of the standard paradigms to describe multifractal scaling, (see [26], [7], [22] for details). Finally, the width of the multifractal spectrum  $f(\alpha_h)$  at  $f = 0$ , given by

$$\Delta\alpha_h = h(-\infty) - h(\infty) = \frac{(\ln(b) - \ln(a))}{\ln 2} \quad (11)$$

is computed to compare the strength of multifractality for different records. Incidentally, the multifractal spectrum can also be estimated using wavelets by the wavelet transform modulus maxima (WTMM) method, [27] which we do not pursue here.

## 4 Results

The log-log plot of the fluctuation functions  $F_q(s)$  vs  $s$  for the wind speed records at all four locations with ( $q = -6, -4, -2, 2, 4, 6$ ) using fourth order polynomial detrending is shown in Fig.3. The generalized hurst exponent  $h(q)$  estimated for ( $q = -6, -4, -2, 2, 4, 6$ ) using Eqn.(5) is shown in Fig.4. One can note that at all locations, the slopes  $h(q)$  decrease as the moment is increased from negative to positive values. For example, the slope decreases from 0.88 when  $q = -6$  to 0.6989 when  $q = +6$  for site 3 which is indicative of multifractal behavior. Recent studies have suggested the choice of constrained randomized shuffled surrogates to reject the null that the observed scaling is due to the distribution as opposed to the temporal structure in the data. Constrained randomized shuffles (surrogates) are similar to traditional bootstrapping without replacement, i.e. each member in the original data is retained in the surrogate. While the temporal structure is destroyed the distribution of the original data is retained in the surrogate realizations. The surrogate realizations are equivalent to uncorrelated noise. A good exposition of the concepts of surrogate analysis can be found in [28]. In the present study, the MFDFA studies are performed on random shuffle surrogates of the original data, to reject the claim that the observed scaling exponent obtained on the original data is due to its distribution and not due to correlations present in it. The results shown in Fig. 5 indicate that for all values of  $q$ , the fluctuations scale as  $F_q(s) \sim s^{0.5}$  which signifies a loss of correlations. In other words, there is a loss of multifractality because the slopes  $h(q)$  are independent of  $q$ . The parameters  $a$  and  $b$  determined under the assumption of a binomial multiplicative process Eqn.(10) is also shown Fig.4. Using the values of  $a$  and  $b$  with Eqns.(5), (9) and (10), the multifractal spectrum at all four locations shown in Fig.6 is computed. The width of the multifractal spectrum is calculated from Eqn.(11).

## 5 Conclusions

In this study, the hourly wind speed records at four geographically well separated locations in North Dakota were examined. The focus was on determining the nature of the correlations and hence, the dynamics in the wind speed data. The power-law decay of the power spectrum suggested the presence of possible long range correlations and self-similar behavior. A number of techniques proposed in the past to quantify self-similarity in a given data have been susceptible to non-stationarity which can manifest itself as polynomial trends. This

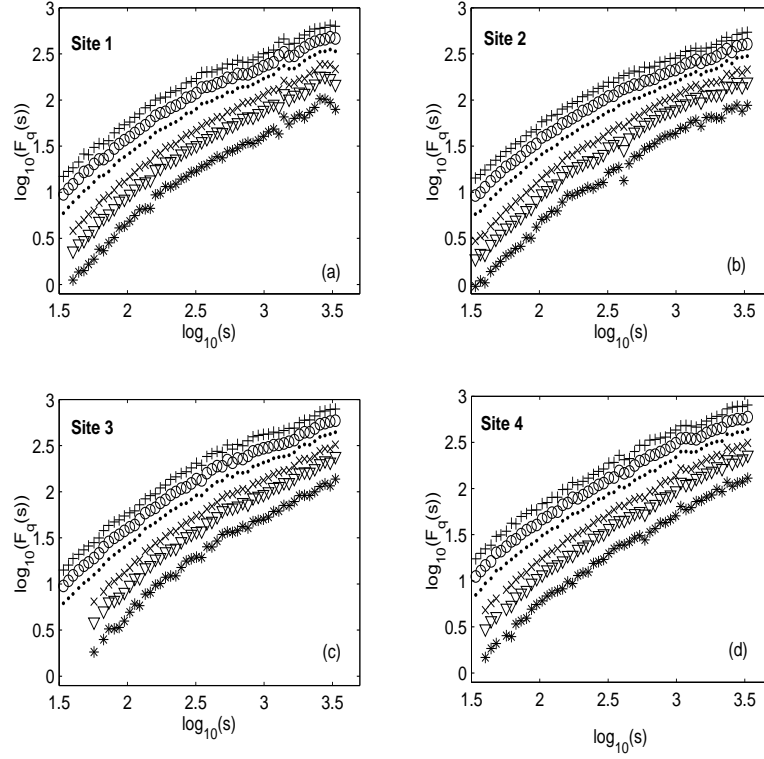


Figure 3: Fluctuation functions at all sites using MFDFA. Symbols used to indicate the various moments are  $q = -6$  (-),  $q = -4$  (inverted triangle),  $q = -2$  (x),  $q = 2$  (.),  $q = 4$  (o),  $q = 6$  (+). The curves are vertically shifted for clarity.

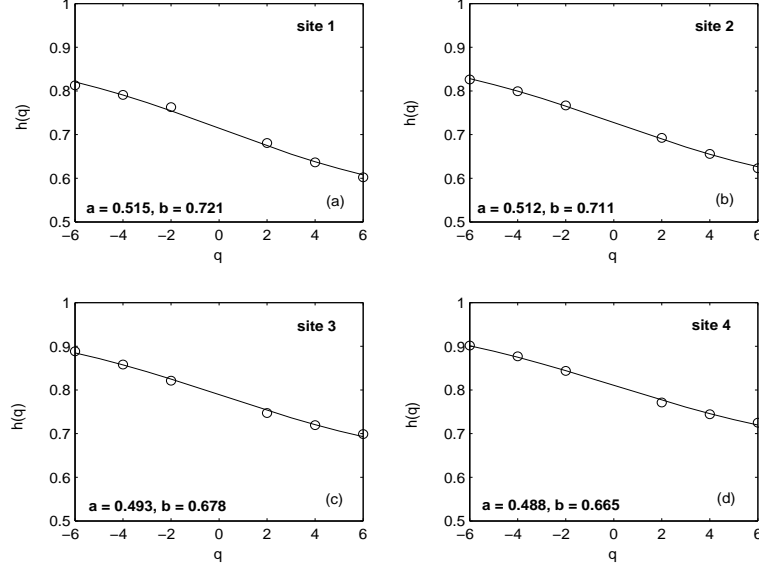


Figure 4: Generalized scaling exponents for wind speed records at the four sites. Circles indicate the actual values of the scaling indices for  $q = -6, -4, -2, 2, 4$  and  $6$ . The solid line represents the fit from the multiplicative cascade model (Eqn.(10)) obtained from a nonlinear least squares procedure. The values of  $a$  and  $b$  are indicated for each site.

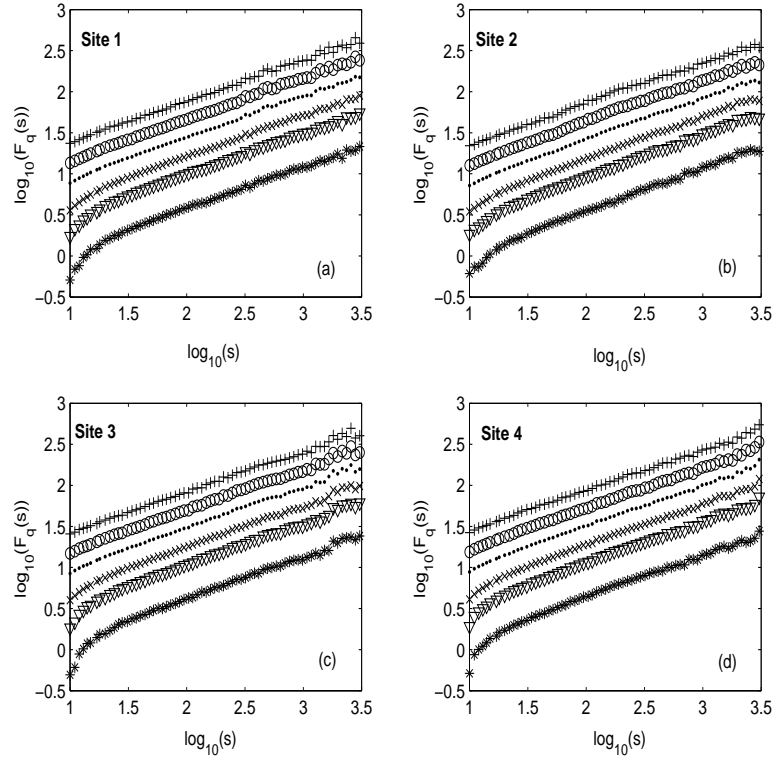


Figure 5: Fluctuation functions at all sites using MF DFA for random shuffle surrogates of the original data. Symbols used to indicate the various moments are  $q = -6$  (-),  $q = -4$  (inverted triangle),  $q = -2$  (x),  $q = 2$  (.),  $q = 4$  (o),  $q = 6$  (+). The curves are vertically shifted for clarity. Note that the curves are parallel to each other with a slope of  $\sim 0.5$ .

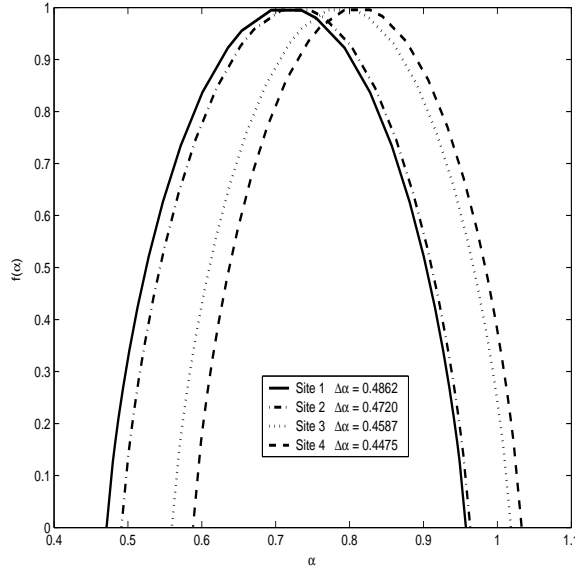


Figure 6: Multifractal spectra of wind speed records at the four sites

motivated us to use a fairly robust technique such as the MFDFA. Analysis using MFDFA elucidated the dependence of  $h(q)$  on  $q$ , which is a hall mark of multifractal processes. The binomial multiplicative cascade model has been widely used to determine the multifractal spectra. The multifractal spectra ( $f(\alpha)$ ) and the widths ( $\Delta\alpha$ ) (assuming a binomial multiplicative cascade) were estimated at the four locations.

Temporal and spatial variations of wind speed are influenced by several factors such as pressure gradient, turbulence, temperature and topography. Our preliminary studies suggest that long term wind speed variations possess a multifractal structure. Moreover, despite the heterogeneities across the spatially separated locations, certain quantitative features such as the spectrum ( $f(\alpha)$ ) and multifractal widths ( $\Delta\alpha$ ) seem to be retained. A deeper investigation of these issues with extensive data sets over wider geographical regions may provide clues towards understanding the nature of long term atmospheric wind speed variations.

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